ATF2 Power Supply Availability Comparison

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### Objective and Conclusions

<table>
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<tr>
<th><strong>Objective</strong></th>
<th>To compare the availability of non-redundant power systems with modular, redundant power systems.</th>
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| **Conclusions** | The SLAC-proposed redundant, modular power systems show significantly greater availability than nonredundant power supplies.  
Hot-swap of the power modules alone does not yield a large availability improvement. Other components must also be redundant.  
ATF2 potentially will use redundant power systems. The ILC definitely will need redundant systems. |
## Glossary of Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
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<tr>
<td>MTBF</td>
<td>Mean time between failures in hours</td>
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<tr>
<td>(MTBF_o)</td>
<td>The increased MTBF in hours that considers equipment operation at lower than rated power levels</td>
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<tr>
<td>(MTBF_R)</td>
<td>The rated MTBF in hours</td>
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<tr>
<td>MTTR</td>
<td>The mean time to repair and recover beam in hours</td>
</tr>
<tr>
<td>(R(t))</td>
<td>Reliability or probability of success with time</td>
</tr>
<tr>
<td>(\lambda, \lambda_o, \lambda_R)</td>
<td>Failure rates in hr(^{-1}). These are the reciprocals of the MTBFs</td>
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<tr>
<td>1/1</td>
<td>One full rated power supply. Rated power = delivered power</td>
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<tr>
<td>1/2</td>
<td>One out of two redundant power supply configuration</td>
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<tr>
<td>2/3</td>
<td>Two out of three redundant power supply configuration</td>
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<tr>
<td>3/4</td>
<td>Three out of four redundant power supply configuration</td>
</tr>
<tr>
<td>4/5</td>
<td>Four out of five redundant power supply configuration</td>
</tr>
</tbody>
</table>
References

1. Diamond Light Source
   Under construction so no empirical Availability data, no reliability studies or analysis
   450 Power supplies are redundant 4/5 configuration.

2. EMI-Lambda, IE Power, Power Ten power supplies  MTBF > 100,000 hours

3. SPEAR 3 power supply operation MTBF >100,000 hours,  MTTR 2 hours

4. Argonne Laboratory, APS, 2000 power supplies, MTBF >> 100,000 hours, MTTR 1 hour

1. MTBF of switchmode power supply or bulk power supply is 110,000 hours based on
   Cherrill Spencer SLAC studies spanning several years

2. MTBF of a single power module is 220,000 hours based on parts count method

3. MTBF PS controllers is 288,889 hours per PAC 2001 reliability paper

4. MTBF cables is 2,600,000 hours per PAC 2001 reliability paper

5. MTTR is 4 hours, 2 hours for repair and 2 hours for beam recovery

6. When redundancy is considered it is Active redundancy

7. No replacement and hot swap replacement during a run are also compared
Availability Improvement By Oversizing and Redundancy

Load = $P_L$

Power Supply
$P_R = 1P_L$
$P_O = 1P_L$
$P_O = 1P_R$

Load = $P_L$

Power Supply
$P_R = 1P_L$
$P_O = 1/2P_L$
$P_O = 1/2P_R$

Load = $P_L$

Power Supply
$P_R = 1P_L$
$P_O = 1/2P_L$
$P_O = 1/2P_R$

Load = $P_L$

Power Supply
$P_R = 1/3P_L$
$P_O = 1/3P_L$
$P_O = 3/4P_R$

Load = $P_L$

Power Supply
$P_R = 1/3P_L$
$P_O = 1/4P_L$
$P_O = 1/4P_R$

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Power Supply
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Power Supply
$P_R = 1/4P_L$
$P_O = 1/5P_L$
$P_O = 4/5P_R$
The general, exponential form of the Binomial Distribution is

\[ R(t) = \sum_{k=m}^{n} \binom{n}{k} \left( \frac{n!}{(n-k)!k!} \right) \left( e^{-\lambda t} \right)^k \left( 1 - e^{-\lambda t} \right)^{n-k} \]

\( \lambda = \text{constant = failure rate} \)

\( m = \text{minimum number of system power supplies needed for operation} \)

\( n = \text{total number of power supplies in the system} \)

A special case occurs when \( m = n \) or when \( m=n=1 \)

\[ R(t) = e^{-n\lambda t} \quad \quad R(t) = e^{-\lambda t} \]
Binomial Expansion 2 out of 3 example

\[ R_{2/3}(t) = \sum_{k=m=2}^{n=3} \left( \frac{n!}{(n-k)!k!} \right) (e^{-\lambda t})^k (1 - e^{-\lambda t})^{n-k} \]

\[ k = 2 \]

\[ \frac{3!}{1!2!} e^{-2\lambda t} (1 - e^{-\lambda t}) = 3 e^{-2\lambda t} (1 - e^{-\lambda t}) \]

3 cases, probability of success, probability of failure

\[ k = 3 \]

\[ \frac{3!}{0!3!} e^{-3\lambda t} (1 - e^{-\lambda t})^0 = 1 e^{-3\lambda t} \]

1 case, probability of success, no failure

\[ R_{2/3}(t) = 3 e^{-2\lambda t} - 2 e^{-3\lambda t} \]
\[ R_{2/3}(t) = 3e^{-2\lambda t} - 2e^{-3\lambda t} \]

**Derivation**

When \( \lambda(t) \) is a function of time

**General form** \( R(t) = e^{-\lambda(t)t} \)

\[
\frac{dR(t)}{dt} = -\frac{d\lambda(t)}{dt} e^{-\lambda(t)t} - \lambda(t) e^{-\lambda(t)t}
\]

\[
\frac{d\lambda(t)}{dt} \text{ is } \ll \lambda(t)
\]

\[
\frac{dR(t)}{dt} = -\lambda(t) e^{-\lambda(t)t} \quad \text{but } e^{-\lambda(t)t} = R(t)
\]

\[
\lambda(t) = \frac{-dR(t)}{dt} \quad \text{If } \lambda \text{ is a constant then the above reduces to } \lambda(t) = \lambda
\]

\[
MTBF(t) = \frac{R(t)}{-\frac{dR(t)}{dt}}
\]
1 power supply where operating power = the rated power

\[ P_O = P_R \]

\[ MTBF_O = MTBF_R \]

\[ \lambda_O = \frac{P_R}{P_O} \lambda_R \]

\[ R_O(t) = e^{-\lambda_O t} \]

\[ A_{O1FR} = \frac{MTBF_O}{MTBF_O + MTTR} \]
For the m out of n case, where \( m \neq n \)

\[ n \text{ quantity of } \frac{m}{n} \text{ rated power modules. Each power module operates at } \frac{m}{n} \text{ rated } P_R \]

\[ P_O = \frac{m}{n} P_R \]

\[ MTBF_O = \frac{P_R}{P_O} \quad MTBF_R = \frac{n}{m} MTBF_R \quad \lambda_O = \frac{m}{n} \lambda_R \]

\[ R_{1/2}(t) = \sum_{k=m}^{n} \left( \frac{n!}{(n-k)!k!} \right) \left( e^{-\lambda_O t} \right)^k \left( 1 - e^{-\lambda_O t} \right)^{n-k} = ne^{-m\lambda_O t} - me^{-n\lambda_O t} \]

\[ MTBF_{m/n}(t) = \frac{ne^{-m\lambda_O t} - me^{-n\lambda_O t}}{mn\lambda_O e^{-m\lambda_O t} - mn\lambda_O e^{-n\lambda_O t}} \]

\[ A_{m/n}(t) = \frac{MTBF_{O_{m/n}}(t)}{MTBF_{O_{m/n}}(t) + MTTR} \]
For the $m=2$ out of $n=3$ case

3-1/2 rated power supplies. Each power supply operates at 2/3 rated $P_R$

$$MTBF_O = \frac{P_R}{P_O}, \quad MTBF_R = \frac{3}{2} MTBF_R, \quad \lambda_O = \frac{2}{3} \lambda_R$$

$$R_{2/3}(t) = \sum_{k=m}^{n} \left( \frac{n!}{(n-k)!k!} \right) \left( e^{-\lambda_O t} \right)^k \left( 1 - e^{-\lambda_O t} \right)^{n-k} = 3e^{-2\lambda_O t} - 2e^{-3\lambda_O t}$$

$$MTBF_{2/3}(t) = \frac{3e^{-2\lambda_O t} - 2e^{-3\lambda_O t}}{6\lambda_O e^{-2\lambda_O t} - 6\lambda_O e^{-3\lambda_O t}}$$

$$A_{2/3}(t) = \frac{MTBF_{O2/3}(t)}{MTBF_{O2/3}(t) + MTTR}$$
MTBF \( \lambda_{1_{\text{sys}}} = \frac{1}{110,000} + \frac{1}{288,889} + \frac{1}{1,300,000} + \frac{1}{2,600,000} \) = 72,960 hours

MTBF \( \frac{\lambda_{1_{\text{sys}}}}{4000} = 18.2 \) hours

MTTR = 4 hours

Availability \( \frac{\lambda_{1_{\text{sys}}}}{\lambda_{1_{\text{sys}}} + \lambda_{\text{MTTR}}} \) = 0.803

Availability is essentially constant
1/2 200 systems & 2000 subsystems
2/3 100 systems & 1000 subsystems
3/4 100 systems & 500 subsystems
4/5 100 systems & 500 subsystems

10 or 5 subsystems per system
Analysis for $1/2$ and $2/3$ Subsystems

$$\lambda_{1-1/2\text{sys}}(t) = \lambda_{BPS} + 10\lambda_{1/2\text{PM}}(t) + 10\lambda_{\text{controller}} + 10\lambda_{\text{transductor}} + 10\lambda_{\text{cabcon}}$$

$$MTBF_{1-1/2\text{subsys}}(t) = \frac{1}{\lambda_{1-1/2\text{subsys}}(t)} \quad \text{MTTR} = 4 \text{ hours}$$

$$\text{Availability}_{1-1/2\text{subsys}}(t) = \frac{MTBF_{1-1/2\text{subsys}}(t)}{MTBF_{1-1/2\text{subsys}}(t) + \text{MTTR}}$$

$$\text{Availability}_{2000-1/2\text{subsys}}(t) = \text{Availability}_{1-1/2\text{subsys}}^{200}(t)$$

Note that availability is a function of time and must be plotted

$$\lambda_{1-2/3\text{subsys}}(t) = \lambda_{BPS} + 10\lambda_{2/3\text{PM}}(t) + 10\lambda_{\text{controller}} + 10\lambda_{\text{transductor}} + 10\lambda_{\text{cabcon}}$$

$$MTBF_{1-2/3\text{subsys}}(t) = \frac{1}{\lambda_{1-2/3\text{subsys}}(t)} \quad \text{MTTR} = 4 \text{ hours}$$

$$\text{Availability}_{1-2/3\text{subsys}}(t) = \frac{MTBF_{1-2/3\text{subsys}}(t)}{MTBF_{1-2/3\text{subsys}}(t) + \text{MTTR}}$$

$$\text{Availability}_{1000-2/3\text{subsys}}(t) = \text{Availability}_{1-2/3\text{subsys}}^{100}(t)$$
Analysis for 3/4 and 4/5 Subsystems

\[ \lambda_{1-3/4\text{subsys}}(t) = \lambda_{BPS} + 5\lambda_{3/4\text{PM}}(t) + 5\lambda_{\text{controller}} + 5\lambda_{\text{transductor}} + 5\lambda_{\text{cabcon}} \]

\[ MTBF_{1-3/4\text{subsys}}(t) = \frac{1}{\lambda_{1-3/4\text{subsys}}(t)} \quad \text{MTTR} = 4 \text{ hours} \]

\[ \text{Availability}_{1-3/4\text{subsys}}(t) = \frac{MTBF_{1-3/4\text{subsys}}(t)}{MTBF_{1-3/4\text{subsys}}(t) + \text{MTTR}} \]

\[ \text{Availability}_{500-3/4\text{subsys}}(t) = \text{Availability}_{1-3/4\text{subsys}}^{100} \]

\[ \lambda_{1-4/5\text{subsys}}(t) = \lambda_{BPS} + 5\lambda_{4/5\text{PM}}(t) + 5\lambda_{\text{controller}} + 5\lambda_{\text{transductor}} + 5\lambda_{\text{cabcon}} \]

\[ MTBF_{1-4/5\text{subsys}}(t) = \frac{1}{\lambda_{1-4/5\text{subsys}}(t)} \quad \text{MTTR} = 4 \text{ hours} \]

\[ \text{Availability}_{1-4/5\text{subsys}}(t) = \frac{MTBF_{1-4/5\text{subsys}}(t)}{MTBF_{1-4/5\text{subsys}}(t) + \text{MTTR}} \]

\[ \text{Availability}_{500-4/5\text{subsys}}(t) = \text{Availability}_{1-4/5\text{subsys}}^{100} \]
Availability Calculation for 4000 \( m/n \) Subsystems

\[
A_{4000-m/n \ subsys}(t) = A_{2000-1/2 \ subsys}(t) \times A_{1000-2/3 \ subsys}(t) \\
\times A_{500-3/4 \ subsys}(t) \times A_{500-4/5 \ subsys}(t)
\]
Availability of 4000 Non-Redundant Vs Redundant Modular PS

No replacement of failed power modules

Nonredundant power supplies
Availability of 4000 Non-Redundant Vs Redundant Modular PS

Hot replacement of failed power modules

Nonredundant power supplies
Availability of 4000 Non-Redundant Vs Redundant Modular PS

- Hot replacement of power modules and redundant bulk PS
- Nonredundant power supplies
Availability of 4000 Non-Redundant Vs Redundant Modular PS

Hot replacement of power modules, bulk PS and PS controllers

Non-redundant power supplies
• The SLAC-proposed redundant, modular power systems show significantly greater availability than the non-redundant systems

• Hot replacement of the power modules alone does not yield a large availability improvement. Other components must also be redundant.

• ATF2 potentially will use redundant power systems. The ILC will definitely need redundant systems